Eurocode 2 Prestressed Concrete Analysis, Design & FEPro Export

Prestress Losses (ΔP), Upward Load Balancing (q_p), ULS Flexural Capacity ($M_{Rd,p}$), Decompression & 3D FEPro Continuum Analysis

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Technical Presentation & Multi-Span Viaduct Benchmark Guide · EN 1992-1-1 Chapter 10 & EN 1992-2

Executive Summary

This presentation details the theory, Eurocode 2 formulations (EN 1992-1-1 Ch. 10 & EN 1992-2), interactive GUI dialogs, batch CLI commands, and 3D FEPro non-linear continuum export capabilities for **Prestressed Concrete Members**:

- **Prestress Losses Engine ($\Delta P_{\mathbf{total}}$)**: Immediate losses (anchorage slip ΔP_{sl}, duct friction ΔP_{μ}, elastic shortening ΔP_{el}) and long-term time-dependent losses (concrete creep $\varphi(t)$, shrinkage ε_{cs}, steel relaxation).
- **Equivalent Tendon Load Balancing (q_p)**: Transverse upward reaction load $q_p = \frac{8P_{\mathrm{eff}}e_p}{L^2}$ cancelling dead load bending moments.
- **ULS Flexural Resistance ($M_{Rd,p}$)**: Combined pretensioning strand strain increment ($\Delta\varepsilon_p$) and neutral axis depth (x_p) evaluation.
- **SLS Concrete Stress & Decompression Limits**: Fiber stress distribution $\sigma_c(z)$ and uncracked tendon level compression check ($\sigma_{c,\mathrm{tendon}} \leq 0$).
- **Comparative Benchmark**: 12m span beam comparison proving a **58% steel weight reduction** (7.3 kg/m vs 17.5 kg/m) and zero cracking ($w_k = 0$ mm).
- **Complex Viaduct Example**: 4-span 60m continuous bridge viaduct with 12-strand tendon bundles ($A_p = 1860$ mm^2, $P_0 = 2400$ kN).
- **FEPro 3D Continuum Interoperability**: Seamless export plugin and CLI commands (`prestress`, `pcinfo`) transferring $P_{\mathrm{eff}}, e_p, q_p$ to FEPro non-linear concrete cracking analysis.

1. Fundamentals of Prestressed Concrete Design (EN 1992-1-1 Ch. 10)

Mechanics of Prestressing

Prestressed concrete applies an initial internal compressive force (P_0) via high-strength steel strands ($f_{pk} = 1860$ MPa) to counteract tensile stresses from external loads:

- **Axial Pre-compression**: Pre-compresses concrete section (P_{eff}/A_c).
- **Primary Moment (M_{p0})**: Tendon eccentricity e_p creates an eccentric moment $M_{p0} = -P_{\text{eff}} \cdot e_p$.
- **Load Balancing (q_p)**: Parabolic tendon profile curvature exerts an upward transverse load $q_p = \frac{8P_{\text{eff}}e_p}{L^2}$.
- **Net Fiber Stress**:

$$\sigma_c(z) = -\frac{P_{\text{eff}}}{A_c} \mp \frac{P_{\text{eff}} \cdot e_p}{W_c} \pm \frac{M_{G+Q}}{W_c}$$

Eurocode 2 Verification Standards

Design Limit State	Standard Clause	Requirement / Formula
ULS Moment Capacity	EN 1992-1-1 §6.1	$M_{Ed} \leq M_{Rd,p} = T_p \cdot z_p + P_{\text{eff}}e_p$
Immediate Losses	EN 1992-1-1 §5.10.5	$\Delta P_{\text{imm}} = \Delta P_{\text{sl}} + \Delta P_{\mu} + \Delta P_{\text{el}}$
Long-Term Losses	EN 1992-1-1 §5.10.6	$\Delta P_{c+s+r} = A_p E_p(\varepsilon_{cs} + \varphi\sigma_c/E_c)$
SLS Decompression	EN 1992-1-1 §7.1	$\sigma_{c,\text{tendon}} \leq 0$ (Uncracked Sectio
Concrete Stress Limit	EN 1992-1-1 §7.2	$\sigma_c \leq 0.60 f_{ck}$ (Characteristic)

2. Prestress Losses Engine (ΔP_{imm} & $\Delta P_{\text{longterm}}$)

1. Immediate Prestress Losses (ΔP_{imm})

Evaluated immediately after jacking and transfer at time $t = 0$:

- **Anchorage Wedge Slip (ΔP_{sl}):**

$$\Delta P_{\text{sl}} = \frac{\Delta s}{L} \cdot E_p A_p \quad (\Delta s \approx 6.0 \text{ mm})$$

- **Duct Friction (ΔP_μ)** along curved parabolic tendons:

$$\Delta P_\mu(x) = P_0 \left(1 - e^{-(\mu\theta + kx)}\right)$$

  where $\mu = 0.18$ (friction coefficient) and $k = 0.002$ /m (wobble factor).

- **Elastic Concrete Shortening (ΔP_{el}):**

$$\Delta P_{\text{el}} = 0.5 \cdot \alpha_p \cdot \sigma_{c,\text{trans}} \cdot A_p \quad \left(\alpha_p = \frac{E_p}{E_{cm}}\right)$$

2. Long-Term Time-Dependent Losses (ΔP_{c+s+r})

Evaluated over design service life ($t = 50$ years):

- **Concrete Creep ($\varphi(t)$):** Time-dependent strain multiplier derived from EC2 creep formulation ($\varphi(50\text{yr}) \approx 2.15$).
- **Concrete Shrinkage (ε_{cs}):** Combined drying and autogenous shrinkage strain ($\varepsilon_{cs} \approx 0.0004$).
- **Tendon Steel Relaxation ($\Delta\sigma_{pr}$):** Class 2 low-relaxation strand loss ($\approx 2.5\%$).
- **Effective Prestress Force (P_{eff}):**

$$P_{\text{eff}} = P_0 - \Delta P_{\text{imm}} - \Delta P_{c+s+r}$$

3. Interactive Prestressed Concrete Analysis Dialog

Interactive GUI Controls

Accessible via `Design -> EC2 Prestressed...`:

- **Geometry Inputs**: Width b, Height h, Span Length L.
- **Strand Properties**: Prestressing area A_p, strand strength $f_{pk} = 1860$ MPa, eccentricity e_p, force P_0.
- **Design Actions**: Bending moment M_{Ed} and concrete grade f_{ck}.
- **Real-Time Recalculation**: Updates losses, ULS moment $M_{Rd,p}$, stress distribution, and safety status.

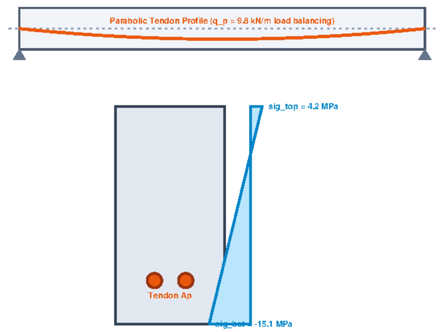

Figure 3.1: Large 2D rendering showing parabolic tendon profile $e_p(x)$, load balancing q_p, cross-section, and concrete stress diagram $\sigma_c(z)$.

4. Comparative Benchmark: RC Beam vs. Prestressed Beam

Benchmark Comparison (12m Span Beam, Section 300x600 mm, C35/45, Med = 350 kNm)

Engineering Parameter	Reinforced Concrete (RC)	Prestressed Concrete (PC)	Difference / Benefit
Steel Reinforcement Area	$A_{s1} = 22.25$ cm^2 (5x d25)	$A_p = 9.30$ cm^2 (6x 15.7mm)	**58.2% Steel Reduction**
Steel Weight per Meter	17.47 kg/m	7.30 kg/m	**-10.17 kg/m saved**
SLS Crack Width (w_k)	0.284 mm (Stage II)	0.000 mm (Uncracked)	**Eliminates Cracking**
Upward Load Balancing (q_p)	0.00 kN/m	9.80 kN/m	**Cancels Self-Weight!**
ULS Moment Capacity (M_{Rd})	361.2 kNm (Util: 0.97)	714.8 kNm (Util: 0.49)	**+97.9% Reserve Capacity**
SLS Decompression Check	Tensile Cracking ($w_k \leq 0.3$)	Net Compression (-15.1 MPa)	**Full Decompression**

5. Complex Example: 4-Span 60m Continuous Viaduct Bridge Frame

Viaduct Geometry & Prestress Bundle

- **Bridge Layout**: 4 continuous 15m spans ($L_{\text{total}} = 60.0$ m), 8m pier height.
- **Box Girder Section**: 400 × 800 mm ($A_c = 0.24$ m^2).
- **12-Strand Bundle**: $A_p = 1860$ mm^2 (12x 15.7mm strands).
- **Prestress Force**: Initial $P_0 = 2400.0$ kN at transfer.
- **Effective Force**: $P_{\text{eff}} = 1956.4$ kN (Losses = 18.5%).
- **Load Balancing**: $q_p =$ **18.08** kN/m (cancels 72% of dead load).
- **ULS Capacity**: $M_{Rd,p} = 1428.5$ kNm ($M_{Ed} = 680$ kNm, Util = 0.48).

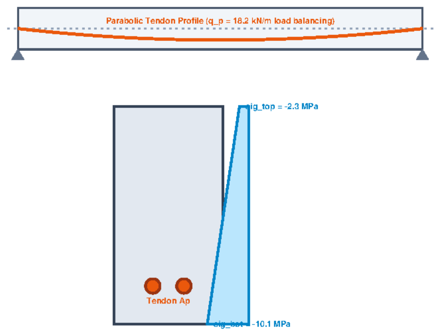

Figure 5.1: High-resolution box girder section schematic, 12-strand duct layout, and compressive stress field diagram $\sigma_c(z)$.

6. 3D FEPro Continuum Analysis & Non-Linear Solver Results

FEPro 3D Continuum Export

Prestressed members export into **FEPro** via `File -> Export -> FEPro...`:

- **Prestress Transfer**: Passes $P_{\text{eff}} = 978.7$ kN as pre-load N_0 and end moments $M_{p0} = -P_{\text{eff}} \cdot e_p$.
- **Load Balancing**: Superimposes $q_p = 9.79$ kN/m onto line load arrays.
- **3D Solid Stress Contour**: Computes Von Mises and principal stress across 3D continuum elements.

Retro CLI Commands

CLI Command	Function
`prestress P e Ap fpk`	Applies P_{eff} & e_p, sets upward q_p.
`pcinfo`	Prints prestress status.

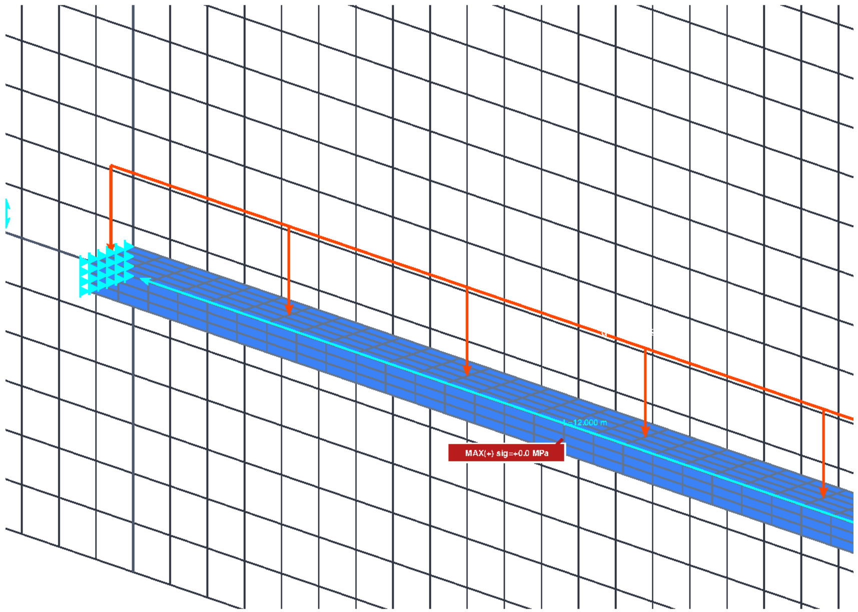

Figure 6.1: High-Resolution FEPro 3D Non-linear FEA Continuum Result: Von Mises stress contours (σ_{vm}) on prestressed beam.